The string solution in SU(2) Yang-Mills-Higgs theory

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Abstract

The tube solutions in Yang - Mills - Higgs theory are received, in which the Higgs field has the negative energy density. This solutions make up the discrete spectrum numered by two integer and have the finite linear energy density. Ignoring its transverse size, such field configuration is the rest infinity straight string.

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At the end of 50-th years W.Heisenberg has been investigate the non-linear spinor matter theory (see, for example, [1], [2]). It is supposed that on the basis one or another nonlinear spinor equation the basic parameters of the elementary particles existing at that time will be derived: masses, charges and so on. The mathematical essence of this theory lies in the fact that the nonlinear spinor Heisenberg equation (HE) (or in the simpler case the nonlinear boson equation like nonlinear Schrödinger equation) has the discrete spectrum of the solutions having physical meaning (possesing, for example, the finite energy). This solutions give the mass spectrum in classical region even. This gave hope that after quantization more or less likely mass spectrum and the charges of the elementary particles would be derive.

Now the string can to arise in Dirac theory with the massive vector field A_μ by interaction 2 magnetic charges with opposite sign [3]. At present time

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the investigations continue along this line and explore not 1-dimensional object (string) stretched between quarks (see, for example, [4]) but 3-dimensional (tube) filled by field (see, for example, [5], [6]). So, for example, a tube of the chromodynamical field and its properties in [5] is considered. But this consideration is phenomenological because a question on the reason of the field pinching isn't affected, also a question on the field distribution in the tube isn't analyzed.

In this article we shown that the Yang - Mills field interacted with Higgs scalar field is confined in tube. In this case the Higgs field have the negative energy density.

In [2] it is showed that the nonlinear Klein - Gordon and Heisenberg equations have the regular solutions. They are the spherical symmetric particlelike solutions numered by integer, i.e. they form discrete spectrum with the corresponding energy value. One would expect (and this will be showed below) that we have in axial symmetric case as well as in spherical - symmetric case the physical interesting (string) solutions with finite energy per unith length.

Finally, we present some qualitative argument in favour of the existence such field configurations (tube, string) according [4]. In QCD vacuum field taken external pressure on the gluon tube. Diameter of such tube will be defined from equilibrium condition between external pressure of the vacuum field and internal pressure of the gluon field in tube. It can be evaluate by minimizing the energy density of such tube which is the difference between the positive energy density of the chromodynamic field and negative energy density of vacuum field in QCD. This diameter R_0 after corresponding calculations is equal:

$$R_0 = \frac{\Phi}{(2\pi^2 B)^{1/4}}, \tag{1}$$

where Φ is a gluon field flux generating quark - antiquark pair, $(-B)$ is the negative energy density of the vacuum chromodynamical field.

We seek a self-consistent solution of the Yang - Mills - Higgs equations system described the Yang - Mills field confined in the tube. We choose Lagrangian in the following form:

$$\mathcal{L} = -\frac{1}{4g^2} F_{a\mu\nu} F_a^{\mu\nu} - \frac{1}{2} \left(D_\mu \Phi \right) \left(D^\mu \Phi \right) - V(\Phi), \tag{2}$$

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where $a = 1, 2, 3$ is $SU(2)$ colour index; $\mu, \nu = 0, 1, 2, 3$ are spacetime indexes; $F_{a\mu\nu} = \partial_\mu A_{a\mu} - \partial_\nu A_{a\mu} + \epsilon_{abc} A_{b\mu} A_{a\nu}$ is the strength tensor of the $SU(2)$ gauge field; $F_{\mu\nu} = F_{a\mu\nu} t^a$, t^a are generators of the $SU(2)$ gauge group; $D_\mu \Phi = (\partial_\mu + A_\mu)\Phi$; $V(\Phi) = \lambda(\Phi^+ \Phi - 4\eta^2)/32$; g, η, λ are constant; Φ is an isodoublet of the Higgs scalar field;

The Yang - Mills - Higgs equations system look by following form in this model:

$$D_\mu F_a^{\mu\nu} = (-\gamma)^{-1/2} \partial_\mu \left[(-\gamma)^{1/2} F_a^{\mu\nu} \right] + \epsilon_{abc} A_{b\mu} F_c^{\mu\nu} = \frac{g^2}{2} \left\{ (D^\nu \Phi)^+ (t^a \Phi) + (t^a \Phi)^+ (D^\nu \Phi) \right\}, \tag{3}$$

$$D_\mu D^\mu \Phi = 2 \frac{\partial V(\Phi)}{\partial \Phi^+}, \tag{4}$$

where γ is the metrical tensor determinant.

We seek the string solution in the following form: the gauge potential $A_{a\mu}$ and the isodoublet of the scalar field Φ we chosen in cylindrical coordinate system (z, r, θ) as :

$$A_{1t} = 2\eta f(r), \tag{5}$$

$$A_{2z} = 2\eta v(r), \tag{6}$$

$$A_{3\theta} = 2\eta r w(r), \tag{7}$$

$$\Phi = \begin{pmatrix} 2\eta\varphi(r) \\ 0 \end{pmatrix} \tag{8}$$

By substituting Eq's(5-8) in Eq's(3- 4) we receive the following equations system:

$$f'' + \frac{f'}{x} = f \left[4 \left(v^2 + w^2 \right) - g^2 \varphi^2 \right], \tag{9}$$

$$v'' + \frac{v'}{x} = v \left[4 \left(-f^2 + w^2 \right) - g^2 \varphi^2 \right], \tag{10}$$

$$w'' + \frac{w'}{x} - \frac{w}{x^2} = w \left[4 \left(-f^2 + v^2 \right) - g^2 \varphi^2 \right], \tag{11}$$

$$\varphi'' + \frac{\varphi'}{x} = \varphi \left(-f^2 + v^2 + w^2 + 1 - \varphi^2 \right), \tag{12}$$

here the dimensionless coordinate $x = r\eta\sqrt{\lambda}$ is introduced; $(')$ means the derivative with respect to x; and the following renaming are made: $g^2 \lambda^{-1/2} \rightarrow$

$g^2, f(x)\lambda^{-1/2} \rightarrow f(x), v(x)\lambda^{-1/2} \rightarrow v(x), w(x)\lambda^{-1/2} \rightarrow w(x)$. We will study this system by the numerical tools. In this article we investigate the easiest case $v = f = 0$. Thus system (9-12) look as:

$$w'' + \frac{w'}{x} - \frac{w}{x^2} = -g^2 w \varphi^2, \tag{13}$$

$$\varphi'' + \frac{\varphi'}{x} = \varphi\left(1 + w^2 - \varphi^2\right). \tag{14}$$

We begin the numerical integration of the assumed equations from point $x = \Delta \ll 1$. For which purpose we expand functions f and φ into a series:

$$\varphi = \varphi_0 + \varphi_2 \frac{x^2}{2} + \cdots, \tag{15}$$

$$w = w_0 x + w_3 \frac{x^3}{3!} + \cdots, \tag{16}$$

after which, substituting in Eq's(13-14) we find that:

$$\varphi_2 = \frac{1}{2}\varphi_0\left(1 - \varphi_0^2\right), \tag{17}$$

$$w_3 = -\frac{3}{4}g^2 w_1 \varphi_0^2. \tag{18}$$

We denote the boundary value $\varphi_0 = \varphi^*$. Then, the numerical investigation shows that there are regions of the boundary values φ^* for which either $\varphi(x) \xrightarrow{x\to\infty} +1$ or $\varphi(x) \xrightarrow{x\to\infty} -1$. It is sufficiently evident that between two this regions there is a boundary value $\varphi_n^*(x)$ caused to the exceptional solution $\varphi_n(x)$ (integer n is the knot number of the function $\varphi_n(x)$). This solutions is separatrix in phase space (φ', φ).

It can to find by succesive approximation method this exceptional solution, wich drop to zero as an exponent by $x \rightarrow \infty$. We notice that in [8] the Eq.(12) with $f = v = w = 0$ has been investigated and the result is analogical achieved her. The spherical symmetric solution of such equation has been investigated in [2] and also was received that only with the discrete boundary value of the wavefunction a particlelike solutions be exist with the finite energy.

The right side of Eq.(13) tend to zero (by $x \rightarrow \infty$) by this φ_n^* value and hence the asymptotical behaviour of $w(x)$ function may be following:

$$w(x) \approx \pm x, \tag{19}$$

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$$w(x) \approx \frac{1}{x}. \tag{20}$$

The numerical investigation show that there are the regions of the parameter g for which $w'(x) > 0$ ($w(x) \approx +x$) and there are the regions of the parameter g for which $w'(x) < 0$ ($w(x) \approx -x$). It is sufficiently evident that there is the exceptional value g^* on the boundary between two such regions caused to the exceptional solution of the system (13 - 14) such that $w(x) \approx 1/x$. Analogously it can to find this exceptional solution $w(x)$ by succesive approximation method. The numerical calculation show that the g^* value may be enumerate by integer m indexed the knot number of the $w(x)$ function.

It can to show that the asymptotical behaviour of $\varphi(x)$ and $w(x)$ functions look as following:

$$\varphi_{mn}(x) \approx \frac{\exp(-x)}{\sqrt{x}}, \tag{21}$$

$$w_{mn}(x) \approx \frac{C}{x} - \frac{Cg^2}{4} \frac{\exp(-2x)}{x^2}, \tag{22}$$

where integers m and n enumerate the knot number of $\varphi(x)$ and $w(x)$ functions respectively. According to this we shall denote the boundary value $\varphi(0)$ and parameter g in the following manner: φ_{mn}^* and g_{mn}^*. The result of numerical calculations on Fig.1,2 are displayed ($w_1 = 0.1$).

The asymptotic behaviour of the $\varphi_{mn}(x)$ and $w_{mn}(x)$ functions as in (21)-(22) results in that the energy density of this fields drop to zero as exponent on the infinity and this means that this tube has the finite energy per unit. It is easy to show that a flux of colour "magnetic" field H_z across the plane $z = const$ is finite.

Thus we can to speak that the Yang - Mills - Higgs theory have the tube solution if the Higgs field have the negative energy density. It is notice that this solutions are not topological nontrivial thread. Ignoring the transversal size of obtained tube we receive the rested boson string with finite linear energy density.

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